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CORRELATION OF TRANSIENT AND FREQUENCY RESPONSES BY USE OF PHASE MARGIN

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ALVIN L. MEINTS

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AND
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CORRELATION OF TRANSIENT AND FREQUENCY RESPONSES BY USE OF PHASE MARGIN

by

Alvin L. Meints
Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of
MASTER OF SCIENCE
IN
ELECTRICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California

1954

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IN

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PREFACE

A linear automatic control system must be stable and yet meet certain specifications of accuracy. In general, both transient and frequency response are needed to guarantee satisfactory designs.

The purpose of this investigation was to show that phase margin can be used to correlate transient and frequency response characteristics, primarily the resonant frequency peak and the transient peak overshoot.

Using the electric analog computer, numerous servo systems were simulated. The resonant frequency peak and transient peak overshoot were plotted versus phase margin for various system gain settings. From the plots approximate equations relating the resonant frequency peak, the transient peak overshoot and the phase margin were developed. Solutions to the equations are generally accurate to within a small percentage of the correct values.

For design purposes, a more useful correlation of resonant frequency peak and transient peak overshoot exists by plotting the two versus a dimensionless parameter $\frac{K_v}{\omega_1}$. If curves for a sufficient number of systems are grouped in families, accurate resonant peak and transient peak values are determined easily with the knowledge of the system gain and the system time constants.

This report has shown that phase margin is the parameter that correlates the resonant frequency peak and the transient peak overshoot. The correlation may be partially ambiguous if the system possesses two or more phase margins of the same value. However, the direct correlation of resonant frequency peak and peak overshoot is still valid.

System gain and gain margin also display a regular variation with phase margin. But the variation is dependent on other system characteristics as well as phase margin.

Phase margin can be a powerful tool to the designer of servo systems. Its use is not limited to certain systems but is applicable to systems of higher order. It is easy to use and gives reliable results.

The author wishes to express his appreciation to Dr. G.J. Thaler, U.S. Naval Postgraduate School, for his assistance, suggestions, cooperation and encouragement in the preparation of this paper.

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TABLE OF SYMBOLS AND ABBREVIATIONS

(Listed in the order of their use in the text)

θ_o	The output or controlled variable.
θ_i	The input signal.
$f(t)$	Time or transient response.
ω	Radians per second.
$F(j\omega)$	Frequency response.
ϕ_M	Phase margin, equal to 180 degrees minus the transfer function phase angle at the frequency at which the gain is zero decibels.
M_p	The $\left\|\frac{\theta_o}{\theta_i}\right\|$ resonant frequency peak.
P.O.	Transient peak overshoot.
$KG(j\omega)$	The system transfer function.
Ang $G(j\omega)$	Phase angle of KG (jw).
δ	System damping coefficient.
ω_n	System natural resonant frequency.
K_v	System gain.
τ	Time constant.
G.M.	Gain margin, $\left[1- KG(jw)\right\}$ at frequency at which Ang G(jw) = -180°.
R	Resistor.
C	Capacitor.

CORRELATION OF TRANSIENT
AND FREQUENCY RESPONSE
BY USE OF PHASE MARGIN

CHAPTER I

INTRODUCTION

In the design and adjustment of linear servo systems the designer must obtain a system that will be stable and will meet certain specifications of accuracy. In general, both transient and frequency response studies are needed to guarantee satisfactory designs.

The performance of a control system is frequently judged on the basis of its response to a transient input consisting of a step function of position. Abrupt disturbances represent important conditions under which the system response is of interest and in which knowledge of system performance on the basis of sinusoidal inputs alone is not sufficient. Because of the ease with which the system response to a transient input may be determined experimentally with limited equipment, it is desirable to be able to check design results against actual system performance.

Although dynamic stability requirements are more easily based on transients following step and ramp disturbances than on the behavior following sinusoidal disturbances of varying frequency, the design of conventional systems to fulfill the majority of performance specifications is more easily accomplished by studying the response to a sinusoidal disturbance of varying frequency. An important reason for the use of the sinusoidal procedure is the relative ease with which

the dynamic properties of an intricate servomechanism can be represented by an experimentally determined frequency function, while representation of all its properties in terms of a differential equation may be nearly impossible.

The effectiveness of the sinusoidal procedure involves correlation between frequency functions and time functions. Correlation to a degree has been based on noting:

(1) salient features of the transient such as speed of response, frequency of oscillations, degree of stability (i.e., peak overshoot), and steady state error, and

(2) resonance peaking of $\left|\frac{\theta_o}{\theta_i}\right|$ at certain frequencies, and the phase angle relating error to output at low frequencies.

The transient response of a system may be determined analytically by the Laplace transformation. But an exact transient analysis for complex systems involves an undue amount of labor. Frequent use is made of dynamic analyzers and analog computors to limit the time required for obtaining a solution to transient and frequency responses.

An approach to determining the transient response of a system is to determine some correlation between the nature of the closed loop response of systems to steady state and transient inputs. Thus, the transient response of a system having a known response to a steady state input can be determined.

If the frequency response F(jw) is known, the transient behavior f(t) in the time domain can be found. Conversely, if the time response f(t) to a unit step input is known, then the response F(jw) in the frequency domain can be found. The responses are related by the integral

equations:

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)\,\epsilon^{t\omega}\,d\omega \qquad (1)$$

and

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)\,\epsilon^{-j\omega t}\,dt \qquad (2)$$

These equations do not permit general explicit solutions. Their relationship may be explained in the following manner. An arbitrary input signal expressed as a function of time can be represented by a Fourier spectrum. The frequency response curves of the total system indicate the extent to which each frequency component in the signal is transmitted by the system, i.e., the change it undergoes in phase and amplitude. The frequency-amplitude curve representing the input signal multiplied by the amplitude transfer curve of the system gives the Fourier spectrum of the output signal. The phase function representing the input signal added to the phase function of the system gives the phase function of the output signal. However, the output signal expressed as a time function may also be expressed by the Fourier synthesis of the component frequencies. Thus, it is reasonable to expect correlation between the frequency response and the transient response curves for the total system.

Since correlation of a kind does exist between the transient response following a step input and the sinusoidal response as given by the frequency functions, the frequency response or loci studies are highly important in the design of servomechanisms.

The primary purpose of this investigation is to show that there exists for various types of servo systems a regular variation of phase margin with the transient peak overshoot P.O. and the resonant frequency peak M_p.

The data was compiled by computative, graphical and analog computer studies.

The primary result is a series of non-dimensional curves for specific transfer functions, these being usable directly for design or gain adjustment purposes. By the use of these curves the designer can tell for an appropriate system whether he can secure adequate stability and still satisfy other specifications such as accuracy. Since steady-state accuracy and transient response are wholly interdependent and fixed once the loop gain function is established, the designer knows immediately if compensation is required.

A secondary prupose of the investigation is to determine an optimum use of phase margin for higher order systems. Present practice generally involves use of M-N contour charts [1] to predict resonant frequency peaks. The phase margin correlation should permit a relatively accurate prediction without resort to the M-N contour charts.

A third objective is to determine a procedure for future experimental study to further develop the phase margin correlation such that all the system requirements of interest may be determined readily from a knowledge of the phase margin. The correlation examined here involves only peak overshoot and resonant frequency peak. Other important characteristics include speed of response, frequency of transient oscillations and steady state error.

CHAPTER II

PREVIOUS INVESTIGATIONS

Existing correlations of the transient and frequent response of linear control systems may be classified as follows:

(1) Qualitative, due to Bode [2], Ferrell [3], Graham [4], Harris [5] and Hall [6].

(2) Analytic, for second and third order systems [7,8].

(3) Quantitative, due to Floyd [7], Dawson [9], and Harris [10].

(4) Empirical, due to Chestnut and Mayer [11], Herr and Gerst [12] and Hunnicutt [13].

A brief discussion of the various methods follows:

1. (a) The criteria of servo system stability utilized in the frequency approached is based on H. Nyquist's [14] theoretical analysis of regeneration in feedback amplifiers. In the stability adjustment of a servo system it was found necessary to provide margins of safety because of aging and other factors. H. W. Bode [2] discussed the theoretical considerations involved in the selection of such margins in the design of feedback amplifiers and introduced the concept of phase and gain margins used extensively by communication engineers.

Phase margin ϕ_M is frequently defined as the value of the angle $[180^\circ - \mathrm{Ang}\, G(j\omega)]$ for the frequency ω at which $KG(j\omega) = 1$. Gain margin is defined as the log modulus of $[1 - KG(j\omega)]$ when the frequency ω is such that the $\mathrm{Ang}\, G(j\omega) = -180^\circ$. Neither ϕ_M nor gain margin is related directly to M_p. Similarly, the mere existence of a ϕ_M of, say, 40° does not insure a satisfactory response.

1. (b) E.B. Ferrell [3], in discussing the application of Bode's treatment to servo systems, stated that a phase margin of between 40 degrees and 60 degrees and a gain margin of 10 to 20 decibels constituted good design practice. Furthermore, Ferrell showed that the phase shift varied with the slope of the gain curve such that 15 degrees of phase shift accompanied a one decibel change of gain per octave.

1. (c) R.E. Graham [4], stated that if the gain margin is sufficient, the phase margin is usually the dominant factor in determining the amplitude of the initial overshoot. The gain margin needed in a particular design depends on the expected variability of the loop transmission. Radar tracking loops usually have gain margins of the order of 15 decibels or more because of the large number of factors which may cause loop gain to vary.

1. (d) H.Harris, Jr.,[5] has stated that as the magnitude of the frequency function $\left|\frac{1}{\omega^k}\right|$ changes, the phase angle approaches $-\frac{k\pi}{2}$ radians for increasing frequencies. If the loop transfer function magnitude decreases as $\frac{1}{\omega^2}$ or faster for a region near unity gain, the phase angle will be near minus 180° or more and the system will be highly resonant. A good loop transfer function will not decrease in magnitude faster than about $\frac{1}{\omega^{1.5}}$ until the magnitude is below about one. Then the resonance peak will be no greater than 1.3.

1. (e) A.C. Hall [6] gave an approximate index of the transient response based on the comparison of frequency response curves and the transient response for the same curves. The presence of peaks in the amplitude response was associated with the complex roots of the characteristic equation. The height of the peak relative to the

flat, low frequency part of the curve is an index of the real part of the root, tending to increase as the real part decreases. The angular frequency at which the peak occurs is an index of the imaginary part of the root, tending to increase as the magnitude of the imaginary part increases. Thus, the frequency of the peak is an index of the frequency of the oscillatory component of the transient response and the height an index of its damping. In numerical terms, if the height of the peak relative to its response at zero frequency is limited to 1.4, then the damping ratio should lie between 0.5 and 0.8, and the angular frequency of the peak should equal the frequency of oscillation to within twenty percent.

2. (a) G.S. Brown and D.P. Campbell [7], and others, have shown that a correlation exists between the maximum or resonant amplitude of the frequency response Mp and the peak overshoot P.O. for various values of the damping ratio δ . As δ decreases, the value of Mp increases and P.O. increases. The magnitudes of the peaks of the amplitude response are measures of the damping of natural modes of oscillation of the transient response. The frequency band over which the amplitude response has a substantially constant magnitude is a measure of the speed of response to transients since a high natural frequency, and therefore a high speed of response, is linked with a high resonant frequency in the amplitude of the sinusoidal response.

For a system having a second order characteristic equation, an exact relationship for Mp as a function of δ has been computed:

$$\frac{\theta_o}{\theta_i} = \frac{1}{-\frac{\omega^2}{\omega_n^2} + 1 + j2\delta\frac{\omega}{\omega_n}} \qquad (3)$$

Differentiating and setting equal to zero:

$$\omega = \omega_n \sqrt{1-2\delta^2} \tag{4}$$

then $$M_p = \left|\frac{\theta_o}{\theta_i}\right|_p = \frac{1}{2\delta\sqrt{1-\delta^2}} \tag{5}$$

It is easily shown that in terms of the same parameters:

$$P.O. = 1 + \frac{\epsilon^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}}}{\sqrt{1-\delta^2}} \sin\left(\tan^{-1}\frac{\sqrt{1-\delta^2}}{\delta}\right) \tag{6}$$

An examination of Mp shows that it becomes excessive for values of $\delta < 0.4$. As a result, widespread practice has been for Mp to be limited to $1.2 < M_p < 1.6$.

2. (b) For the third order system, no simple relationship exists between system parameters and the coefficients of the factored cubic. Charts relating the coefficients of the general cubic

$$a_3 x^3 + a_2 x^2 + a_1 x + 1 = 0 \tag{7}$$

to the values of α, δ and ω_n in the factored cubic have been prepared by Y.J. Liu and L.W. Evans .

$$\left(\frac{\alpha}{\delta\omega_n} x + 1\right)\left(\frac{x^2}{\omega_n^2} + \frac{2\delta}{\omega_n} x + 1\right) = 0 \tag{8}$$

The use of the charts require appreciable calculations since the coefficients a_1, a_2 and a_3 are not related in any simple way to actual system parameters. Charts relating system parameters to system performance have been made up by H.M. James [8]. However, no correlation of P.O. with the system parameters was made.

3. Analytic methods for correlating steady state and transient response based on the Laplace or Fourier transform have been developed. One method is that of G.F. Floyd described by Brown and Campbell [7]. Floyd's method is based on an approximate solution

of the inverse transform of the known frequency function. The method is easier to use than any exact method for systems greater than the fourth order and is accurate to better than 5 per cent. More recently H. Harris, Jr., M.J. Kirby, and E.F. Van Arx [9], and C.H. Dawson [10] have published methods of developing transient response curves from frequency response.

4. (a) Another method of determining transient response makes use of a series of charts requiring no detailed calculations. These charts relate the open-loop response to either a sinusoidal input signal or to a step input signal.

H. Chestnut and R.W. Mayer [11] have developed such charts for a multitude of systems of a certain character, namely Type I with the third, fourth, or fifth order characteristic equation.

4. (b) D. Herr and I. Gerst [12] have correlated the responses of a limited number of systems possessing minimum band-width. Herr and Gerst did not graph their correlations but presented them in tabular form.

The correlations just described are limited in that they apply only to systems having prescribed open-loop attenuation characteristics and are not general.

W.R. Hunnicutt, Jr. [13] measured gain margin, phase margin and transient response for non-linear systems subjected to amplifier saturation. Hunnicutt plotted ϕ_M versus various system parameters but did not plot Mp or P.O. versus ϕ_M and did not arrive at any successful correlation.

CHAPTER III

PROCEDURE

By reference to Fig. 1, it is seen that ϕ_M is a function of the corner frequencies, the slope of the loop transfer frequency curve and the system gain. The loop transfer function uniquely defines all of the properties of the linear servomechanism. However, for each such transfer function, there is an infinite variety of physical assemblies of servo systems having this transfer function. Hence only the mathematical form of the transfer function was considered here rather than particular physical configurations.

Various systems of the second, third and fourth order and of Types I and II were investigated with system gain the parameter to be varied. This is equivalent to the design problem of adjusting system gain to give desired performance. For example, take the transfer function:

$$KG(j\omega) = \frac{K_v\,(j\omega T_2 + 1)}{j\omega(j\omega T_1 + 1)(j\omega T_3 + 1)} \tag{9}$$

The characteristic equation is:

$$-j\omega^3 + \left(\frac{1}{T_1} + \frac{1}{T_3}\right)(j\omega)^2 + \left(\frac{1}{T_1 T_3} + \frac{K_v T_2}{T_1 T_3}\right) j\omega + \frac{K_v}{T_1 T_3} = 0 \tag{10}$$

The system gain term changes both the system damping and natural frequency.

A schematic diagram of the equipment used to measure the peak frequency response and the peak overshoot is shown in Fig. 2 and equipment is listed in Appendix A.

A loop transfer function was first plotted on the Bode or decibel-log ω plot. Various values of system gain were selected such that the phase margin was varied from about 20 degrees to 70 degrees. Mp was determined by plotting appropriate points on M-N contour charts. P.O. overshoot was

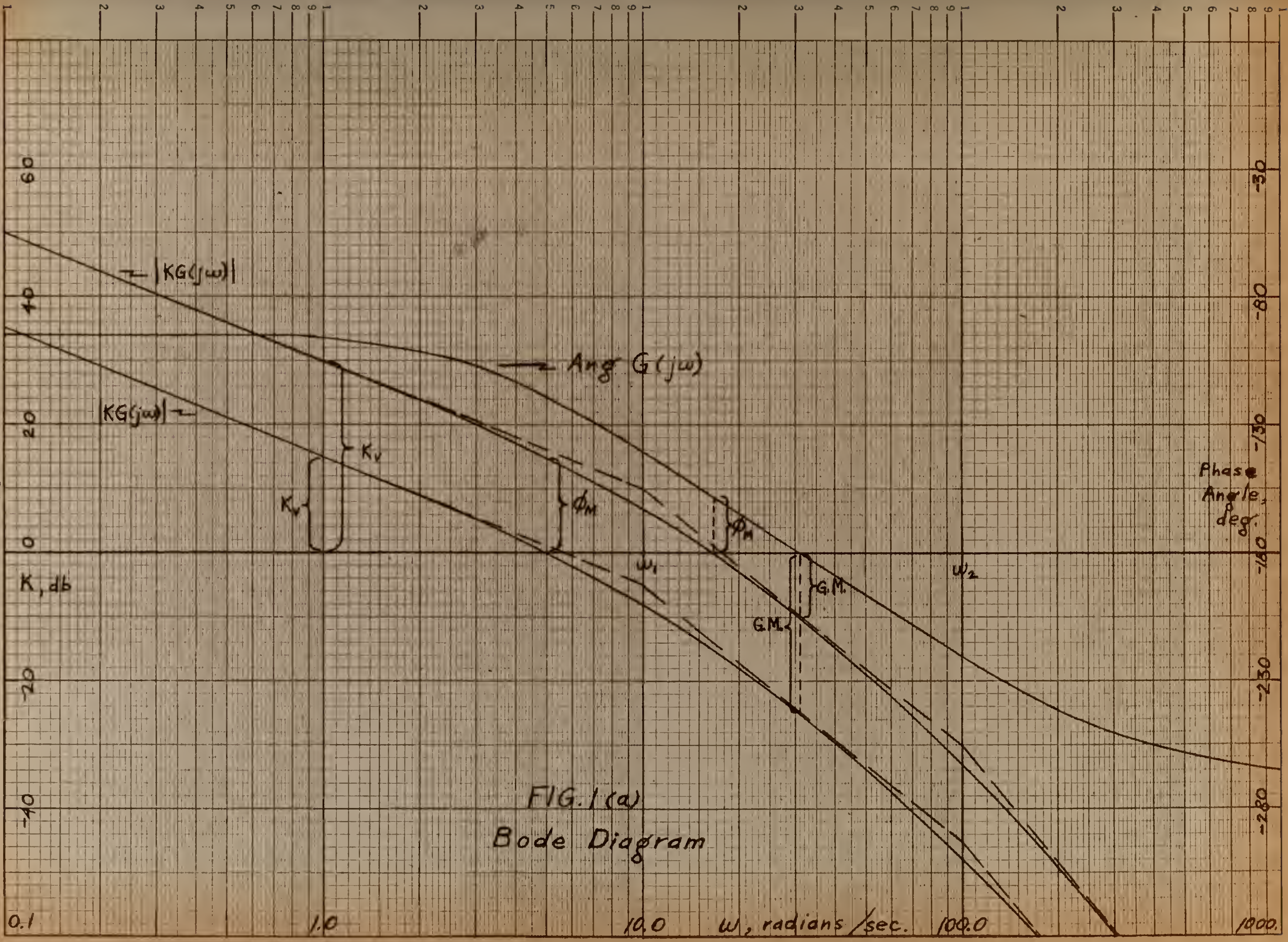


FIG. 1(a)
Bode Diagram

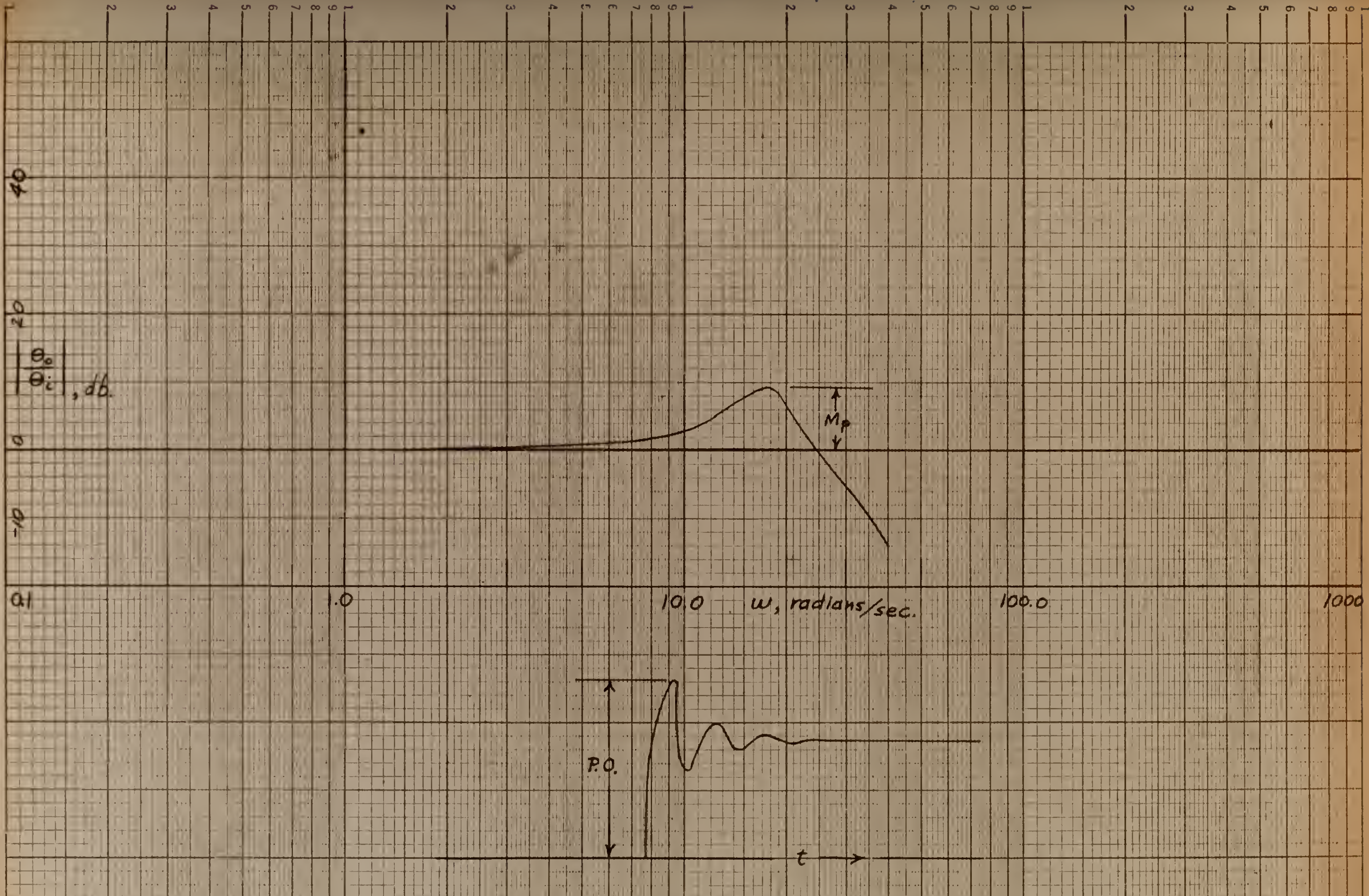


FIG. 1(b)
M_p and P.O.

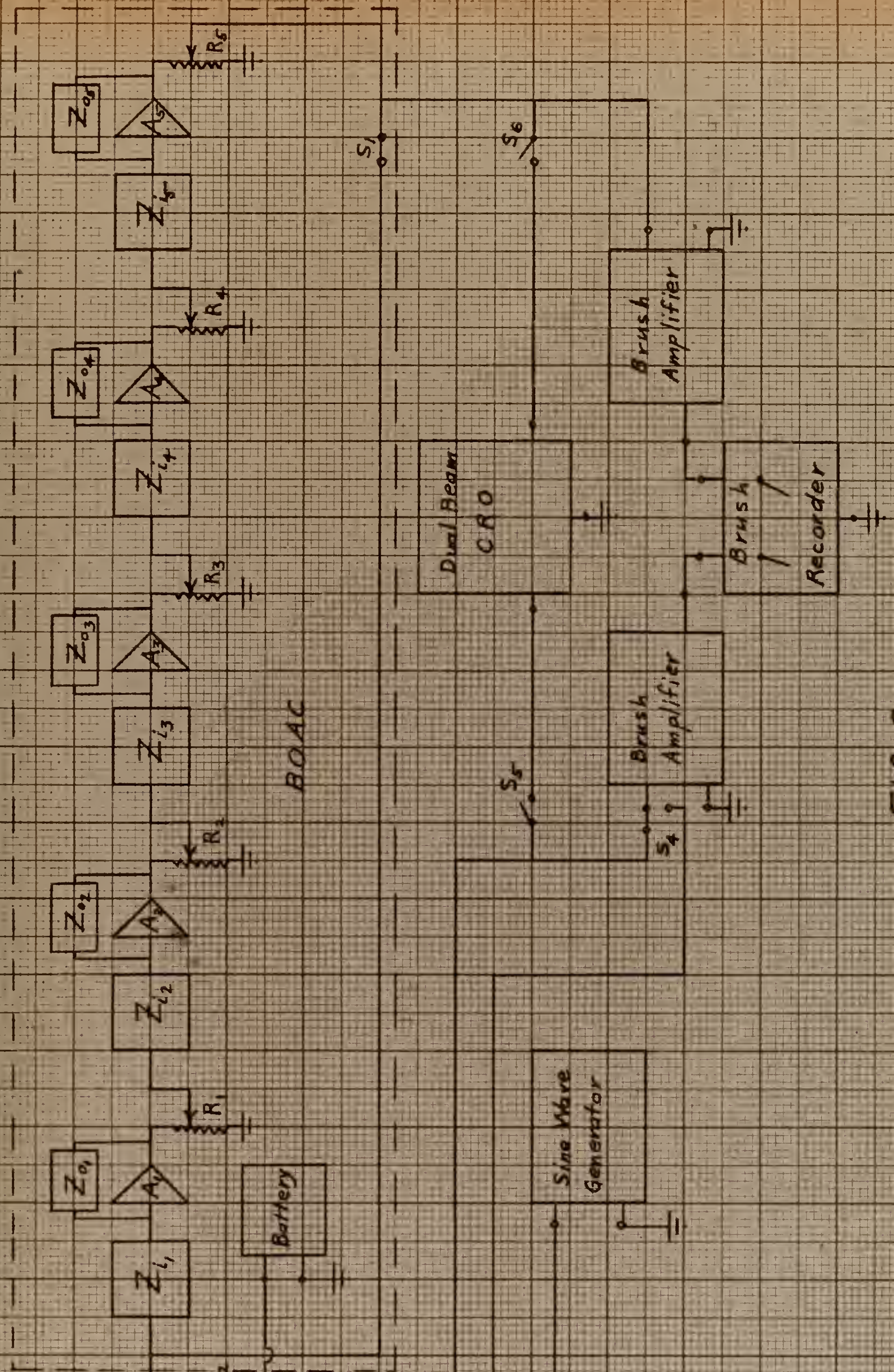


FIG 2
Schematic Diagram of Experimental Set-Up

calculated for at least one point for all systems examined that were of third order or less.

The basic element in the system was the analog computer. A Boeing Analog Computer, BOAC, was available and this was used. Automatic balance units were not available but drift did not give trouble except for high gain systems. In these circumstances, voltage scaling was an adequate measure.

The sine wave generator used was the Mod.202A Hewlett-Packard Low Frequency Generator. At low frequencies the irregularity of the wave shape was quite apparent but since the components contributing to this erratic wave shape were of a much higher frequency order, and very small in amplitude, they did not affect the response of the low pass systems.

The step input was supplied by 45 volt battery installed in the BOAC. The input and output signals were recorded by Brush Magnetic Pens.

For the experimental tests, the R-C components used in the computer were IRC resistors, having ± one per cent tolerance and a temperature coefficient of .025 to .065. Sprague Vitamin Q Capacitors and Cornell-Dublier Metal Cased Dykanol paper capacitors were used.

The components were measured to four significant figures. It was assumed that their values while in use did not change. Experimental values of P.O. for low gain systems not using voltage scaling differed from calculated values by 0.1 to 2 percent. On higher gain systems deviations as high as 7 per cent were recorded.

The measurement of Mp on the Brush recorder was not so good. Since the resonant magnification was frequently 3 times or more, the initial gain setting of the Brush amplifier was so small that it could not be set and measured with sufficient accuracy. Hence, the values of Mp used in this report are those determined on the M-N contour charts.

CHAPTER IV

RESULTS AND DISCUSSION

The significant data taken is recorded in Tables I and II in Appendix B. This data has been plotted in two different manners:

(1) Mp and P.O. versus ϕ_M , Figs. 3, 5, 7, and 9, and

(2) Mp and P.O. versus $\frac{K_v}{\omega_1}$, a nondimensional parameter, Figs. 4, 6, 8, and 10.

The similarity of the curves on the first type of plot may be quite surprising. The sole marked variation is in those systems in which phase lag or lead compensation is inherent or has been intentionally added. This occurs whenever two different system gain settings can produce the same ϕ_M. Even here, though, it is seen that a similarity exists in that the higher gain setting always leads to a larger Mp and P.O. for the same ϕ_M. Also, for the conditionally stable systems, it is apparent that as stability and ϕ_M decrease the curves of P.O. and Mp become indistinguishable from those of systems that are absolutely stable.

An approximate linear relationship between ϕ_M and P.O. is:

$$\log_{10} P.O. = 0.259\left(1 - \frac{\phi_M}{75^\circ}\right) \qquad , 20^\circ \leq \phi_M \leq 60^\circ \qquad (11)$$

This empirical formula is accurate to better than 10 percent for ϕ_M's near 20 or 60 degrees and gives better than 5 percent accuracy for the region near 40 degrees. The looped back portion of the curves (increasing ϕ_M with increasing system gain) was neglected in the preceeding statement but in most cases the empirical equation will still be accurate to within 10 percent.

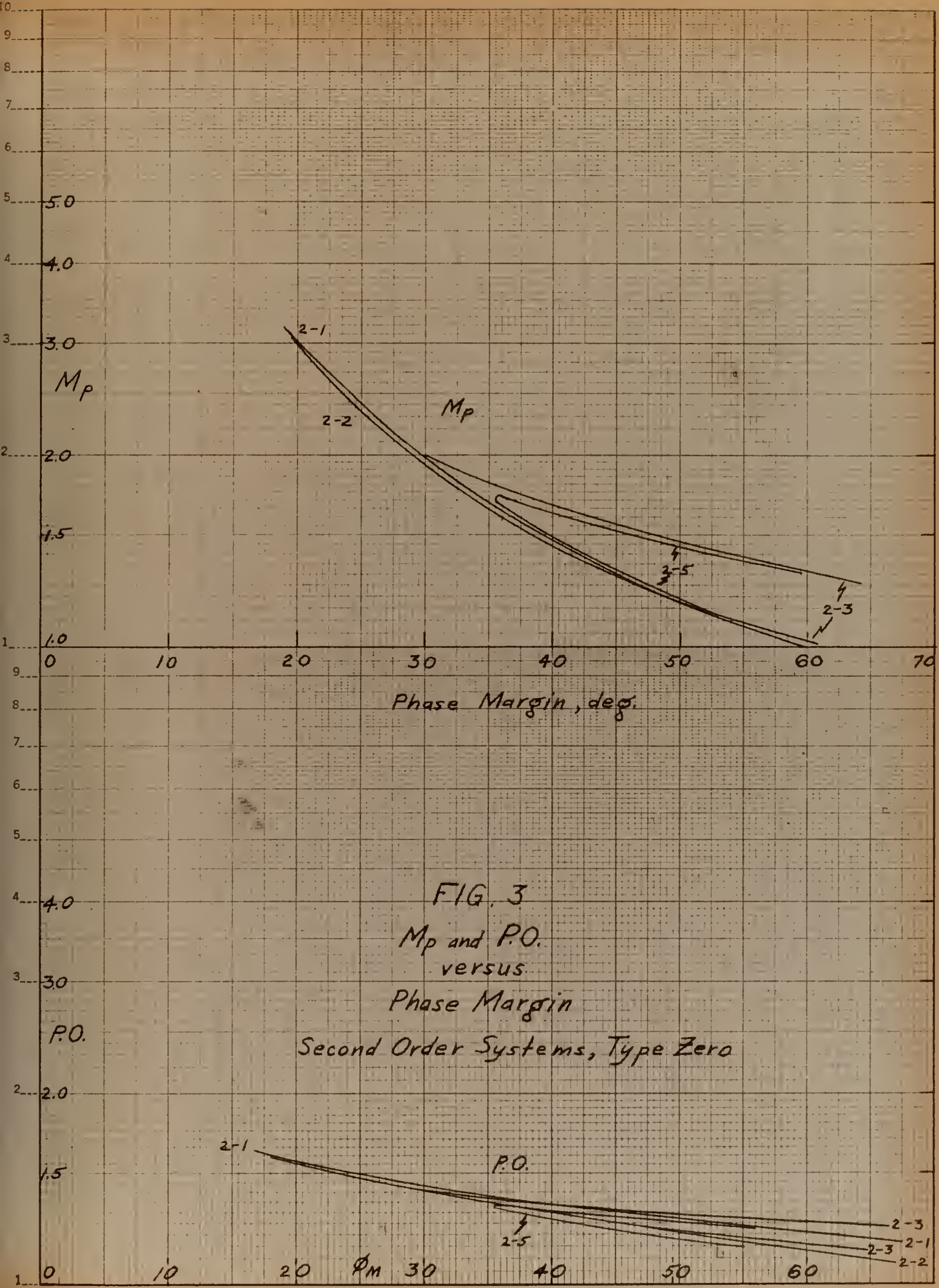


FIG. 3

Mp and P.O.
versus
Phase Margin
Second Order Systems, Type Zero

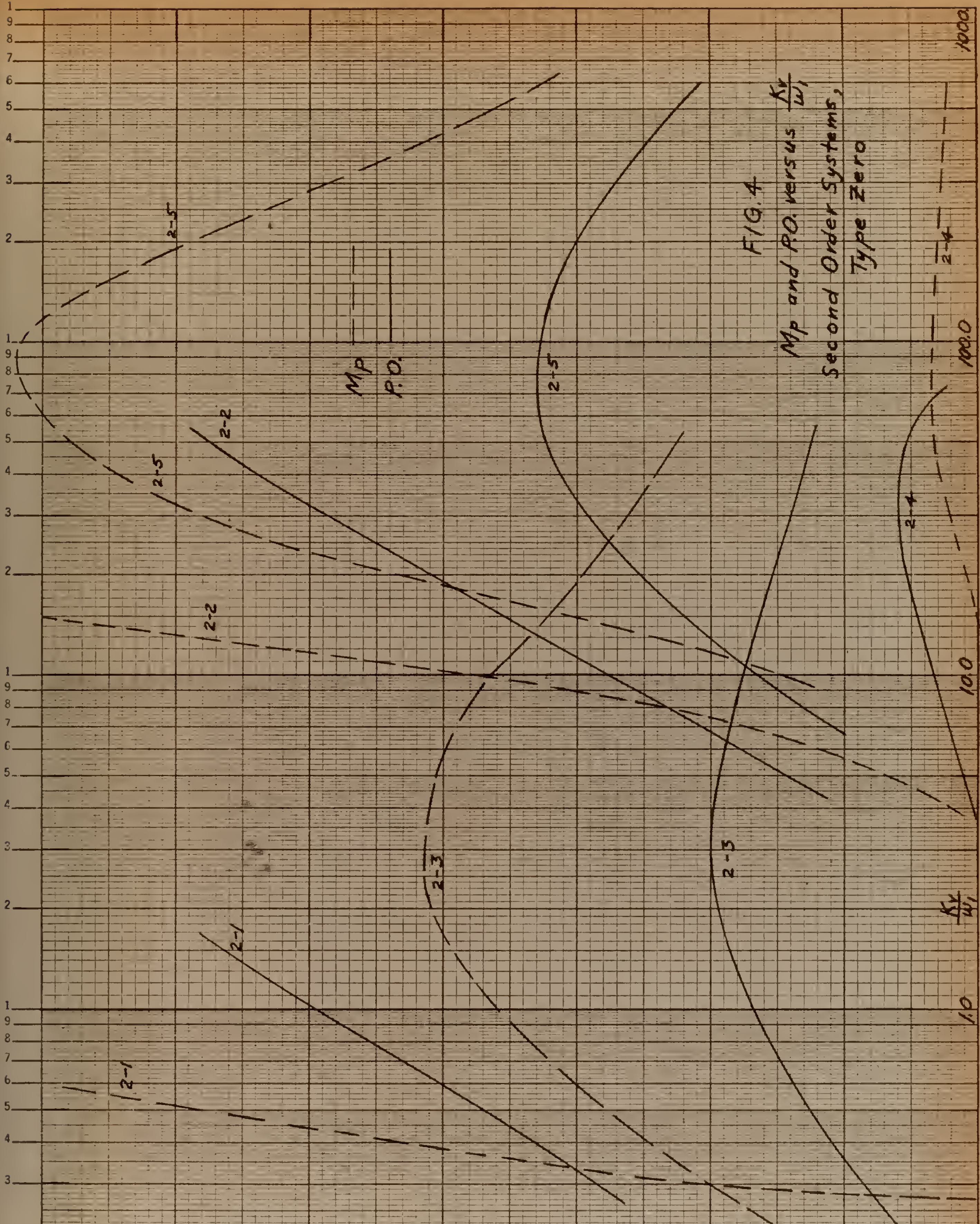


FIG. 4

M_p and P.O. versus $\frac{K_v}{\omega_1}$

Second Order Systems, Type Zero

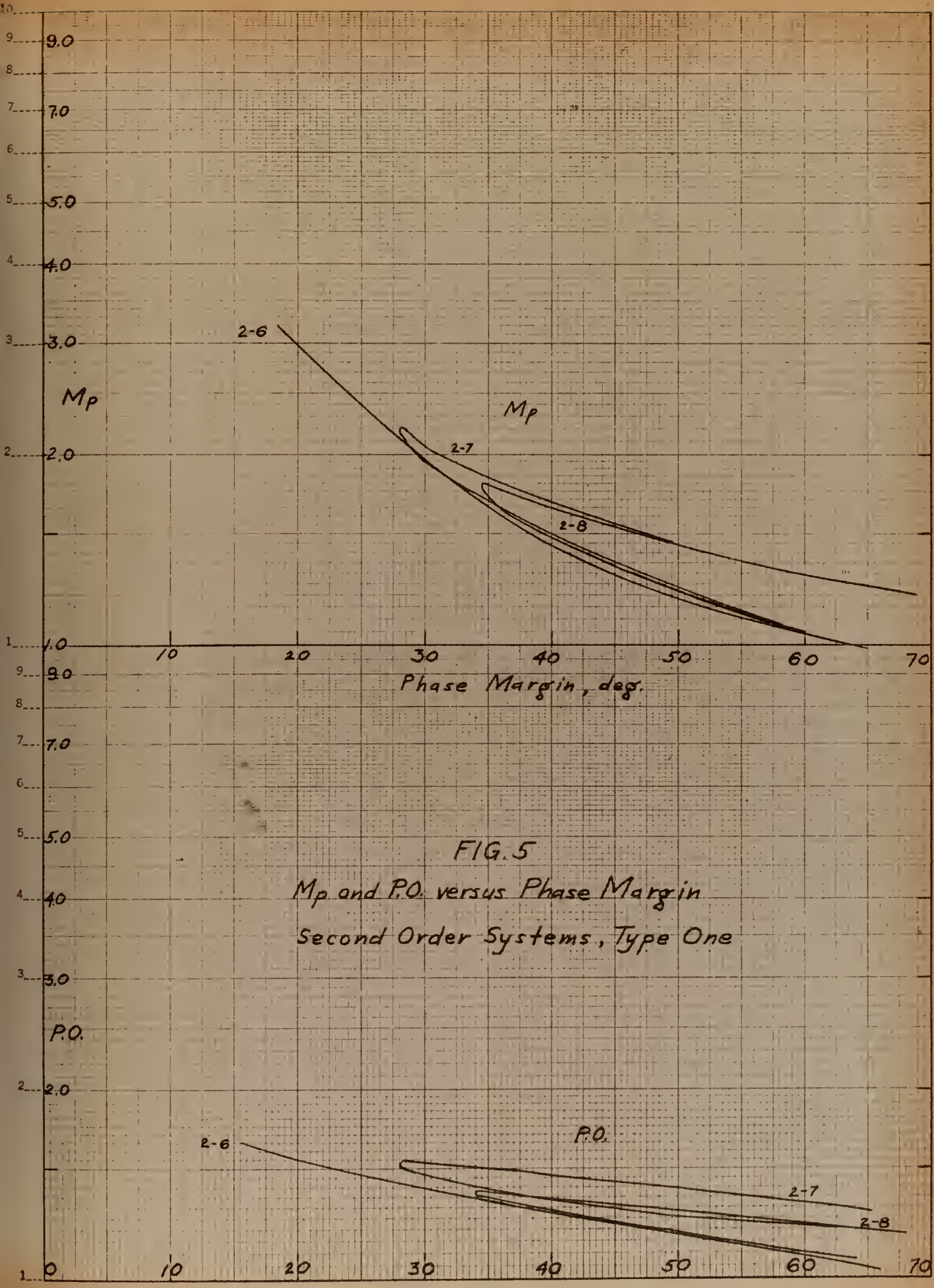


FIG. 5

Mp and P.O. versus Phase Margin

Second Order Systems, Type One

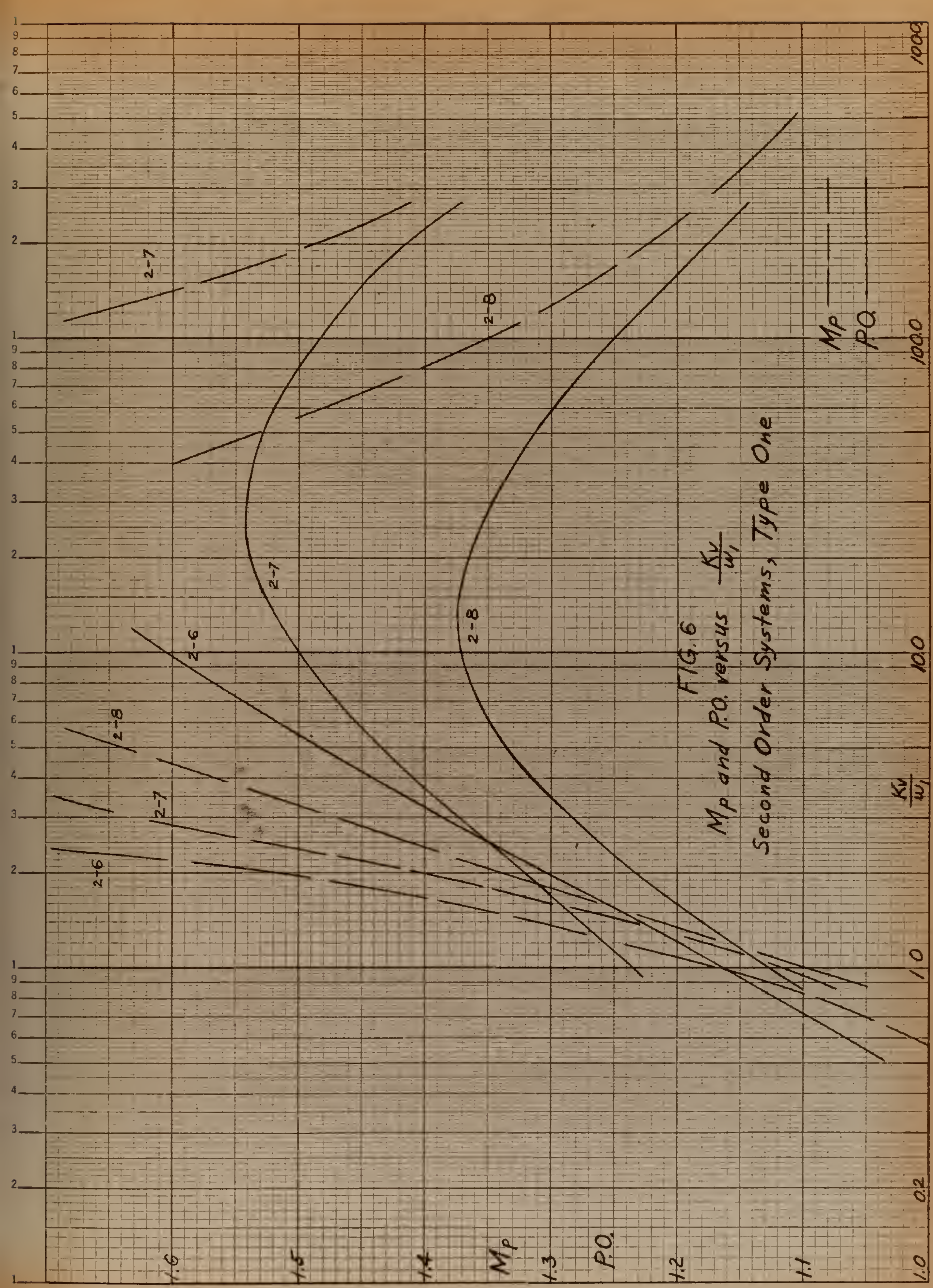


FIG. 6

M_p and P.O. versus $\frac{K_v}{\omega_1}$

Second Order Systems, Type One

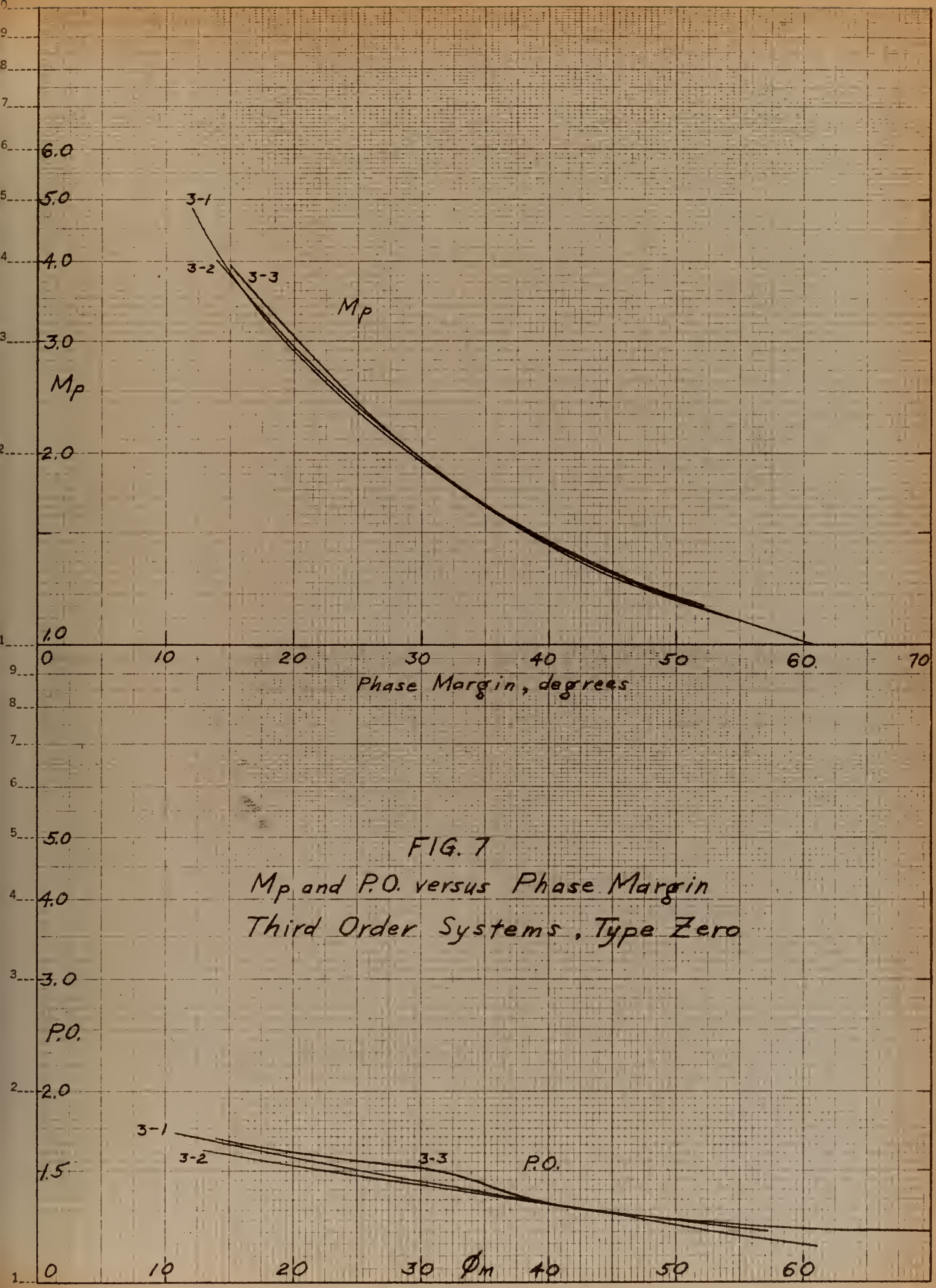


FIG. 7

Mp and P.O. versus Phase Margin

Third Order Systems, Type Zero

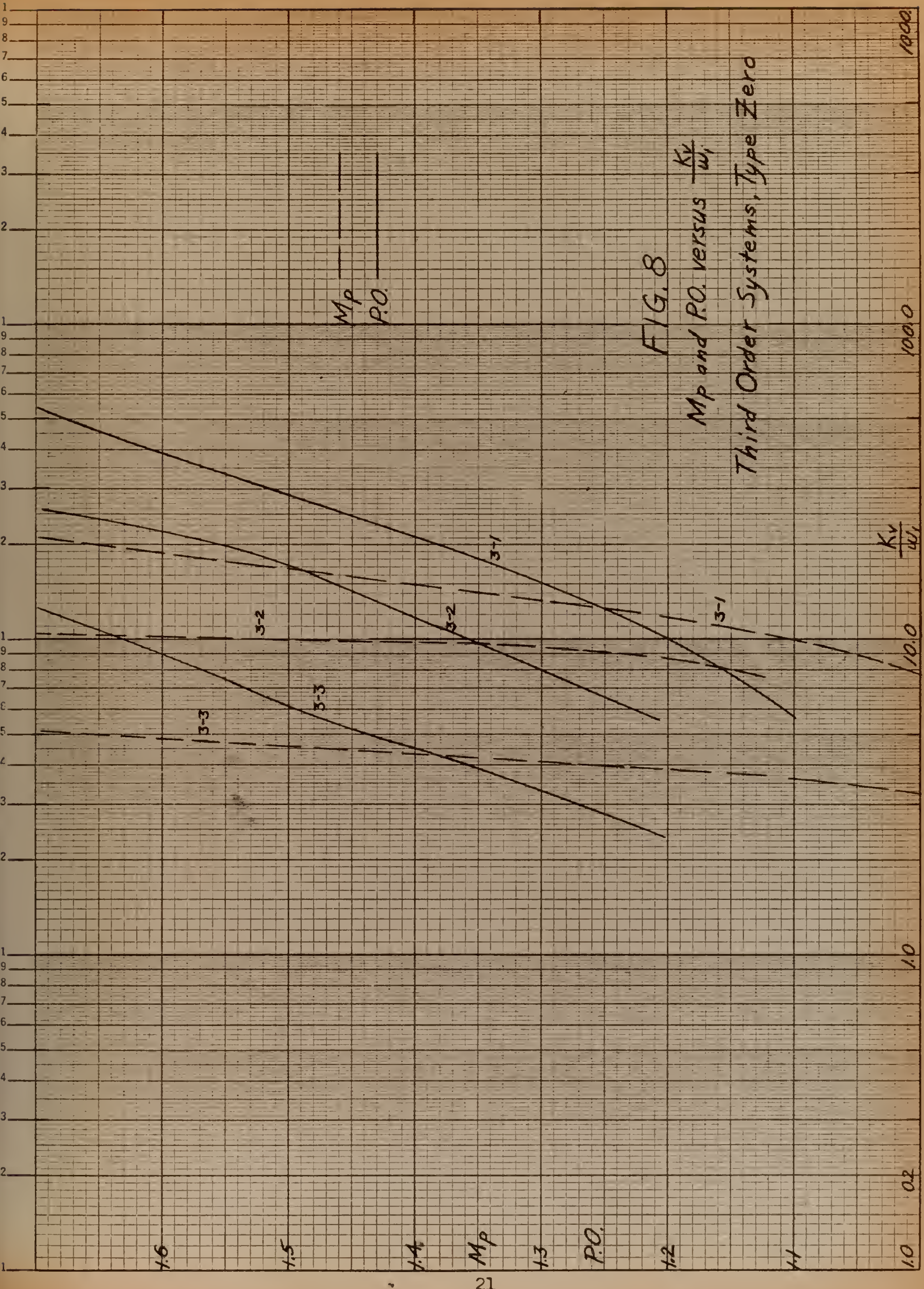


FIG. 8

Mp and P.O. versus $\frac{K_v}{\omega_1}$

Third Order Systems, Type Zero

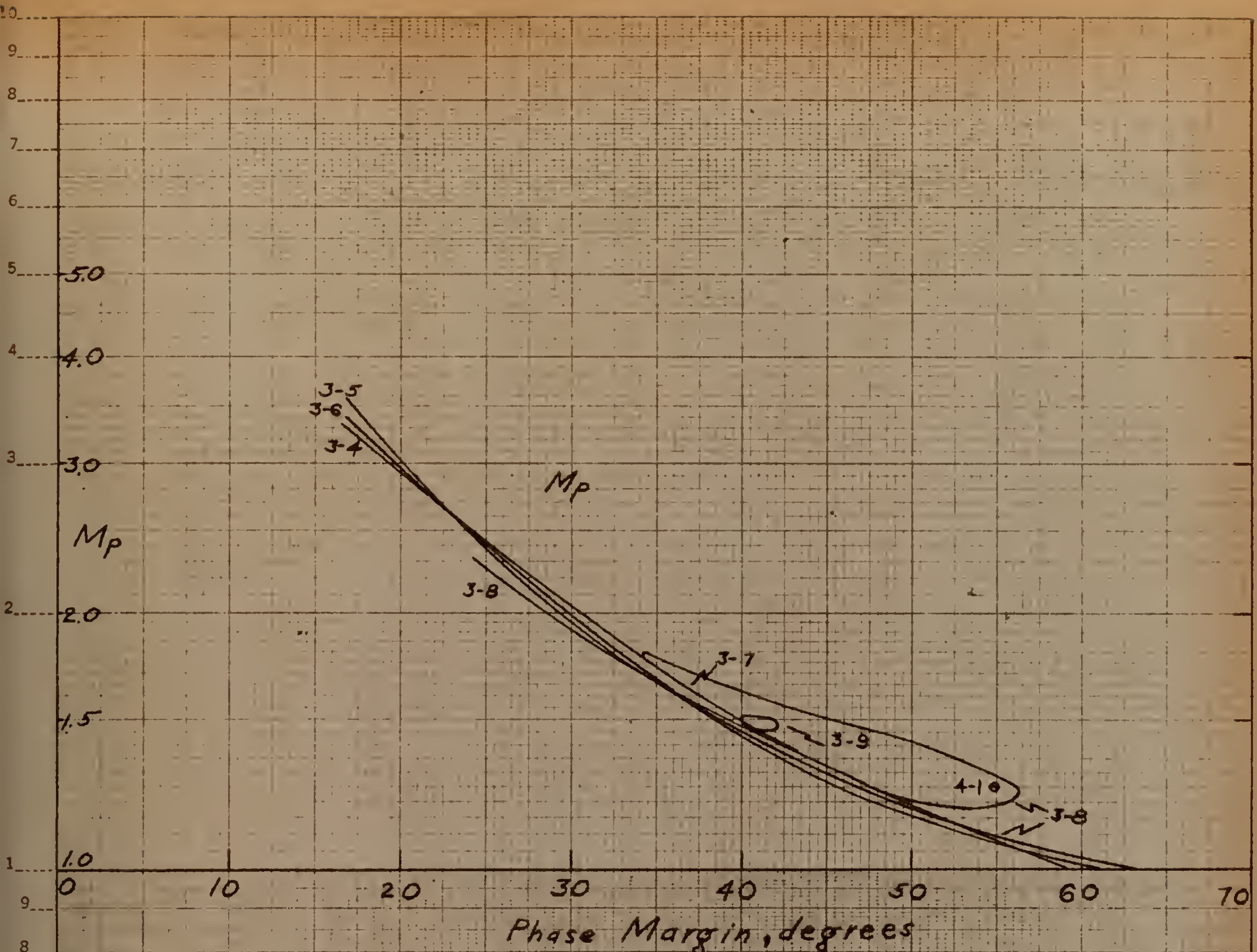
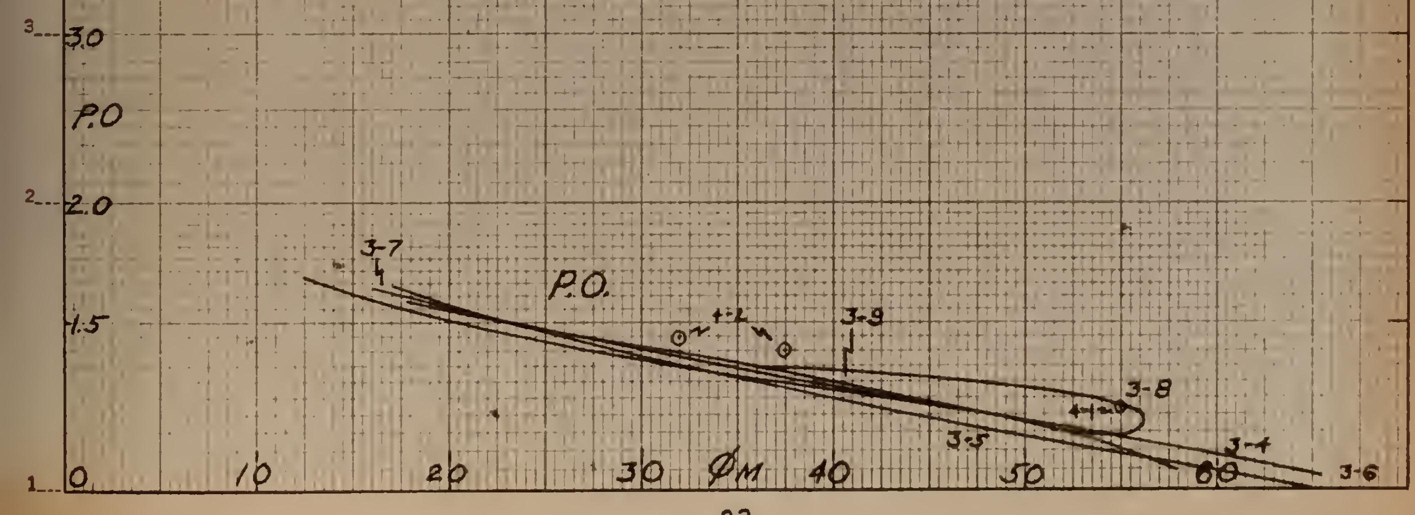


FIG. 9

M_p and P.O. versus ϕ_M

Third Order Systems, Type One



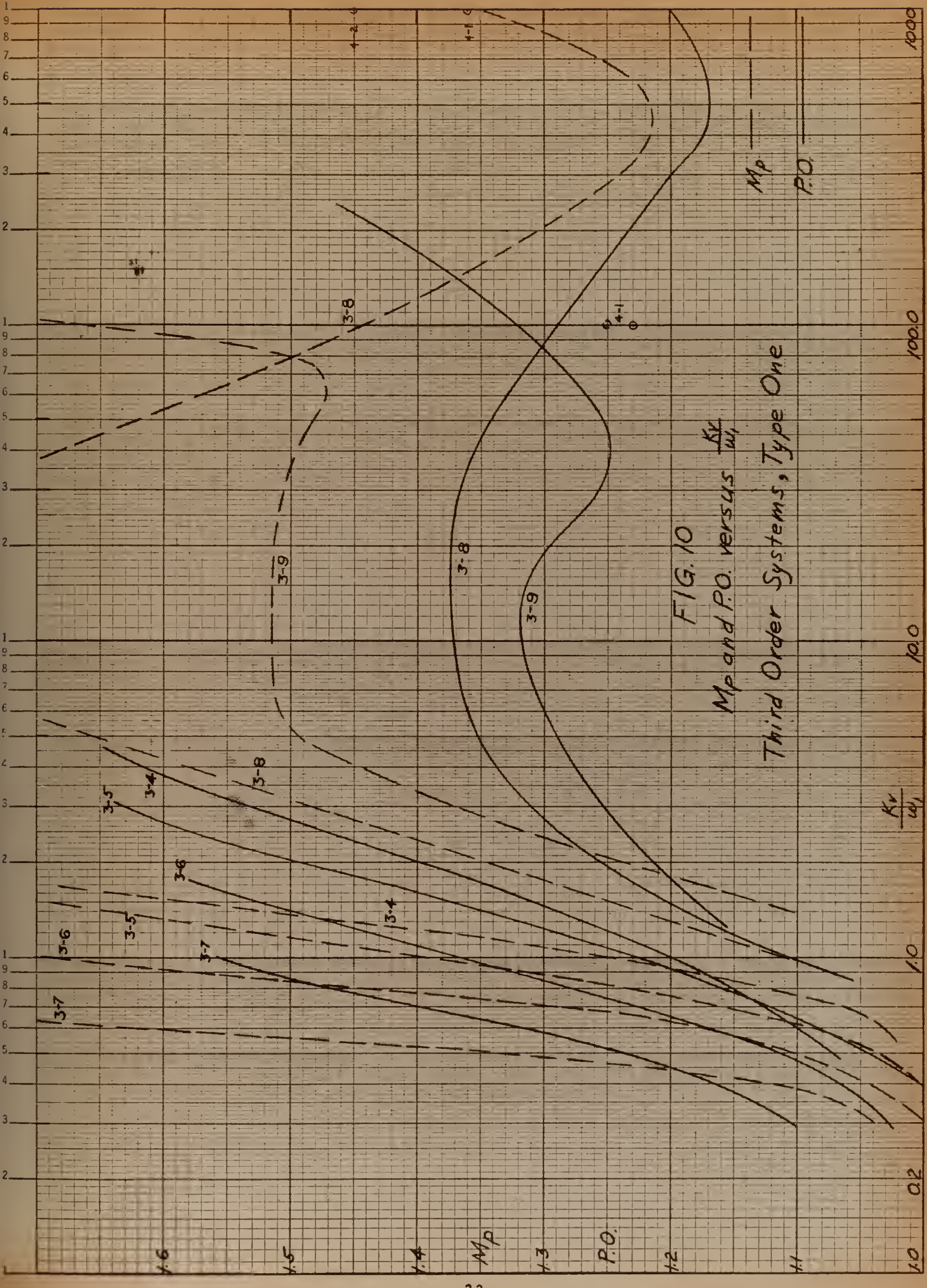


FIG. 10

M_p and P.O. versus $\frac{K_r}{\omega_1}$

Third Order Systems, Type One

Similarly an approximate linear relationship between ϕ_M and P.O. is:

$$\log_{10} M_p = 0.435\left(1 - \frac{\phi_M}{62.5^\circ}\right) \qquad , 40^\circ \le \phi_M \le 60^\circ \tag{12}$$

Again neglecting the folded back portion of the $\phi_M - M_p$ curves (increasing ϕ_M with increasing system gain), this empirical equation is accurate to within 4.5 percent at 40 degrees and 5.0% at 60 degrees phase margin; these ϕ_M limits include the range of greatest interest.

With these two simple equations a designer, knowing ϕ_M only, can predict with better than 10 percent accuracy the Mp and P.O. to be expected. If the ϕ_M is taken in the range where it is increasing with increasing system gain, these equations give optimistic results, the error being greater as the band-width of this characteristic increases.

It is possible to fit empirical equations more accurately to the mean M_p-ϕ_M and P.O.-ϕ_M curves but the improvement in accuracy doesn't warrant the additional complexity involved. Should a more exact expression be desired, an inverse transcendental or a hyperbolical curve such as

$$\phi_M = A \tanh\left(B \log_{10} M_p + C\right) + D \tag{13}$$

should be tried.

The constants A, B, C and D are used to shape the curve [15].

In Fig. 11, Mp versus P.O. is plotted. For the systems examined in this report, it is seen that Mp versus P.O. does have a general relationship. Again a linear empirical formula is used to describe this correspondence.

$$\log_{10} M_p = 2.015 \log_{10} P.O. - 0.087 \tag{14}$$

For the systems of this report, this equation is accurate to about 15 percent. However, for most systems covered it is accurate to within 10 percent. We

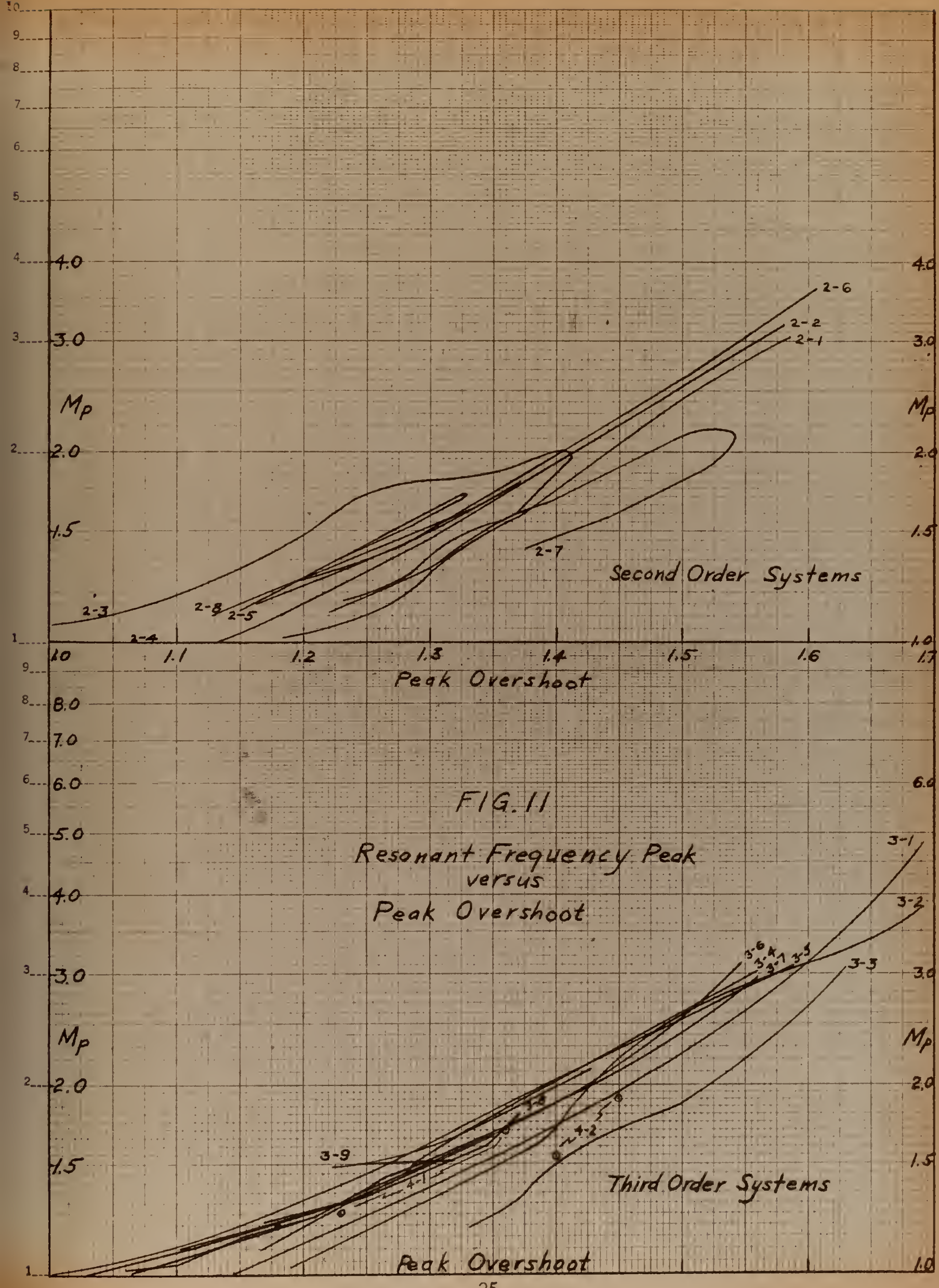


FIG. 11

Resonant Frequency Peak
versus
Peak Overshoot

now have a simple relationship between Mp and P.O. such that if Mp or P.O. is determined experimently, the other is known to within 10 or 15 percent.

Having observed these correlations of Mp and P.O. with ϕ_M and Mp with P.O., an analytic correlation would seem desirable and possible. Taking the expressions for Mp and P.O., equations (5) and (6) in Chapter II, a relationship between P.O., Mp and δ can be derived. However, the expression in the form of a series in δ and Mp does not converge rapidly and so is of little practical aid.

The question arises as to whether the empirical relations derived from Fig. 3 to 10 are useful for other and perhaps more complex systems likely to be encountered. In Table III, Appendix B are listed data for the systems analyzed by Herr and Gerst [12]. For example, taking a system listed under Case III, having a fifth order differential characteristic equation:

$$\phi_M = 43.1^\circ \quad , M_p = 1.4 \quad , P.O. = 1.35$$

$$\log_{10} M_p = 0.435\left(1 - \frac{43.1}{62.5}\right) = 0.135 \quad , M_p = 1.365$$

$$\%\ Error = \frac{.035}{1.4} \times 100 = 2.50\,\%$$

$$\log_{10} P.O. = 0.259\left(1 - \frac{43.1}{75}\right) = 0.11 \quad , P.O. = 1.29$$

$$\%\ Error = \frac{.06}{1.35} \times 100 = 4.45\,\%$$

If the Mp is known, using the empirical relation for P.O.:

$$\log_{10} M_p = 2.015 \times .13 - .087 = 0.175 \quad , M_p = 1.497$$

$$\%\ Error = \frac{.097}{1.4} \times 100 = 6.93\,\%$$

To take an example of a fourth order equation listed under Case IV:

$$\phi_M = 60^\circ \quad , M_p = 1.05 \quad , P.O. = 1.1$$

$$\log_{10} M_p = 0.435\left(1 - \frac{60}{62.5}\right) = 0.0174 \quad , M_p = 1.04$$

$$\%\ Error = \frac{.01}{1.05} \times 100 = 0.95\,\%$$

$$\log_{10} P.O. = 0.259\left(1 - \frac{60}{75}\right) = 0.52 \quad , P.O. = 1.128$$

$$\%\ Error = \frac{.028}{1.1} \times 100 = 2.54\,\%$$

$$\log_{10} M_p = 2.015 \times .041 - .087 = -0.0045 \quad , M_p = 0.9$$

$$\%\ Error = \frac{.06}{1.05} \times 100 = 5.81\,\%$$

By the use of two simple empirical formulas, values of Mp, P.O. or ϕ_M can be determined by knowing any one of the three. Usually these values are accurate to within 10 percent or better.

Generally the designer is given a set of specifications from which he can determine the required steady state frequency response and the transient response to a step input. Frequently certain elements of the servomechanism are fixed as part of the system and cannot be varied. Also, the load or driven element is often a fixed component of the system. With the fixed components plus a knowledge of servo system elements, the designer can determine the remainder of the components necessary to obtain a suitable open-loop characteristic. It is a simple matter to draw the phase angle curve especially if a tangent template is available. Once the time constants of the system are fixed, ϕ_M is changed only by changing system gain. By the use of the empirical equations (11) and (12) the designer has, with very little labor, a reasonable check as to whether system gain and dynamic stability are compatible for the system he has selected. If families of curves, such as Figs. 3, 5, 7, and 9, are available he has an accurate check on Mp and P.O. Furthermore, if families of curves, such as are indicated in Figs. 4, 6, 8, and 10 are available, it is not necessary to draw the phase angle curve, since in Figs. 4, 6, 8, and 10 Mp and P.O. are plotted versus the dimensionless parameter $\frac{K_v}{\omega_1}$. Of course, the attenuation characteristic of the transfer function must be similar to one of the families since $\frac{K_v}{\omega_1}$ versus Mp or P.O. varies widely with the various open-loop characteristic curves.

With the minimum effort the designer is able to tell approximately by the use of empirical equations, or exactly by the use of families of curves, if specifications of stability and accuracy are satisfied or if

compensation is required. The use of the empirical equations is general and not restricted to any particular system. The equations may be optimistic in the region previously noted, but that region is readily apparent from the open-loop attenuation characteristic. The error in the region of increasing phase margin with increasing gain is proportional to the band-width of the compensation. However, if either Mp or P.O. are known, the other may be computed by equation (14) with the same degree of accuracy as stated before.

Thus a regular variation of Mp and P.O. with ϕ_M does exist and may be used to correlate the steady-state and transient characteristics of a system.

CHAPTER V

SUGGESTIONS FOR FURTHER INVESTIGATIONS

In this report phase margin has been used to correlate only two of the steady-state and dynamic properties of a system. Other properties of interest are time of response to a step or ramp input, time for transient to settle within prescribed error limits, the frequency of transient oscillation, the frequency at which the resonant peak Mp occurs, and steady state accuracy.

That a regular change in system gain accompanies a change in ϕ_M is shown in Fig. 12. However, the amplitude of the gain and its slope differs a great deal with the various systems. This is, in part, dependent on the presence of a zero in the right half of the complex plane. The ratios of the time constants or poles of the system also influence the system gain versus ϕ_M relationship.

In Fig. 13 the ratio of gain margin divided by the lowest corner frequency $\frac{G.M.}{\omega_1}$, is plotted versus ϕ_M. Again a regular variation of gain margin with phase margin is apparent; but like system gain, the gain margin slope and magnitude depends on other parameters as well as ϕ_M.

The effect of the spacing of corner frequencies (or time constants) on ϕ_M should be further investigated. If the ratio is much greater than ten (or one cycle on the log ω plot), the larger corner frequencies have little effect on ϕ_M. This is equivalent to the correlation of Y. Chu [16] and J.G. Truxal [17] showing that performance may be determined primarily by a complex pair of poles, simple poles near the orgin, and the zeros in the complex plane.

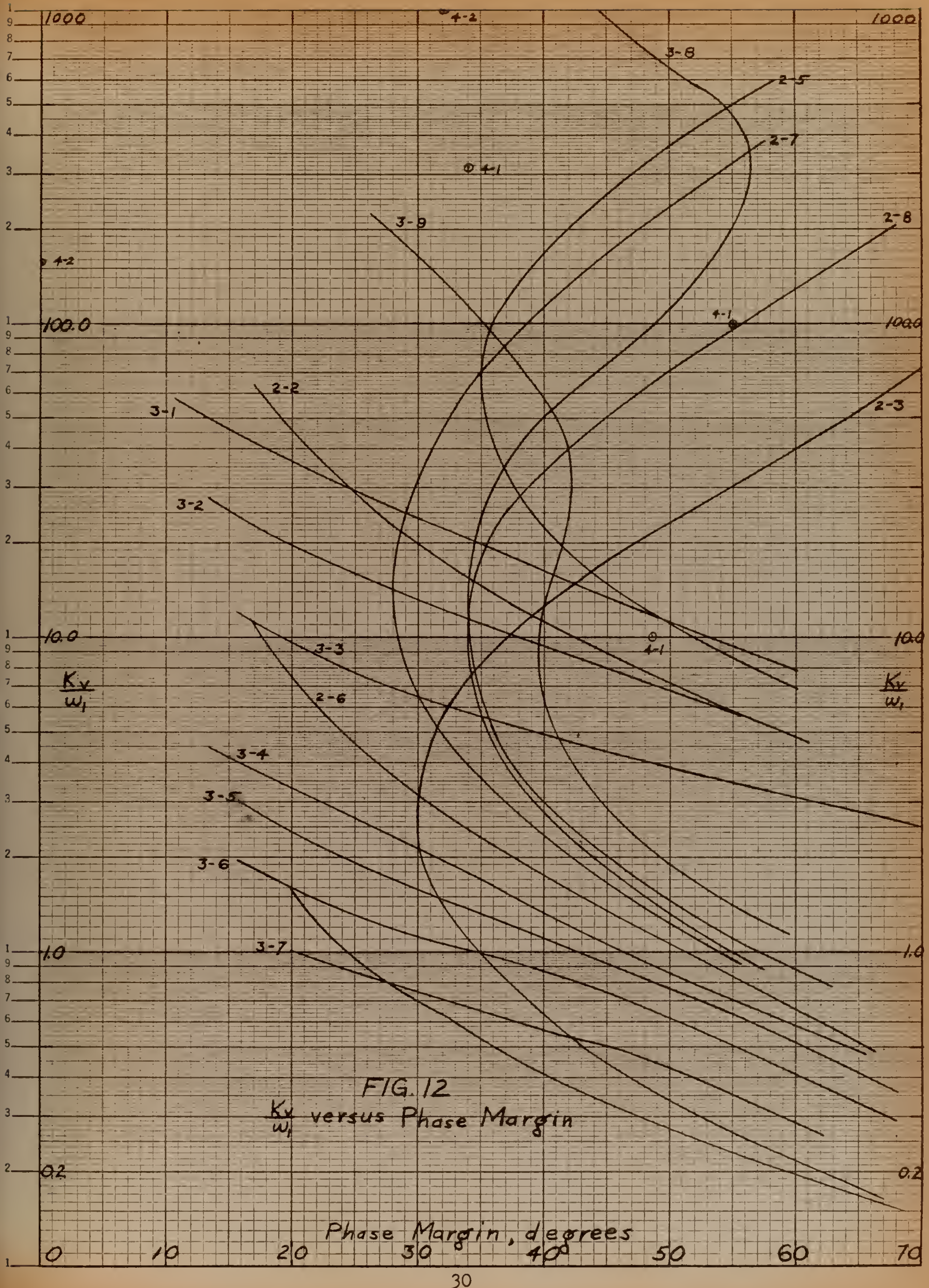


FIG. 12

$\frac{K_v}{\omega_1}$ versus Phase Margin

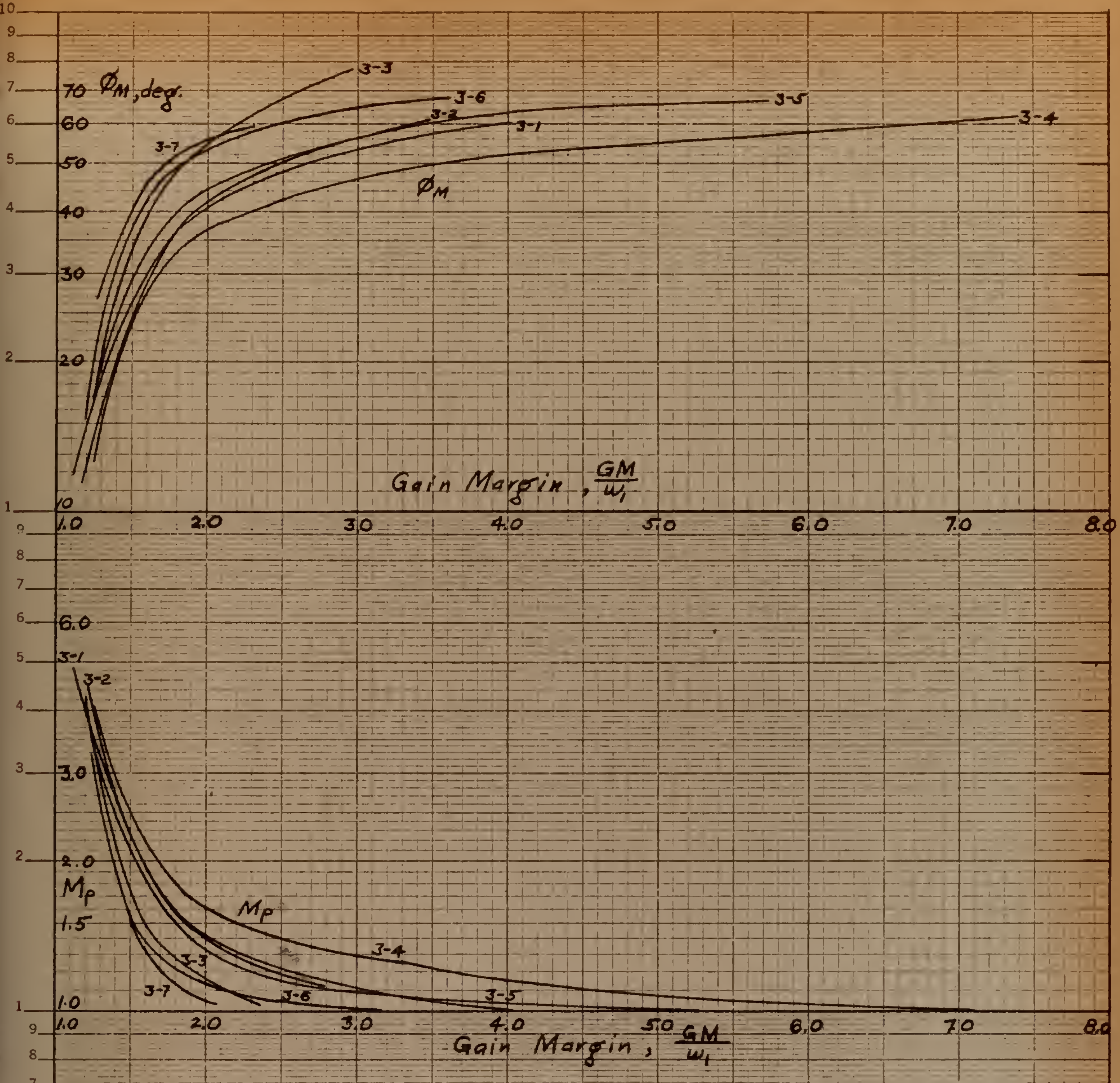
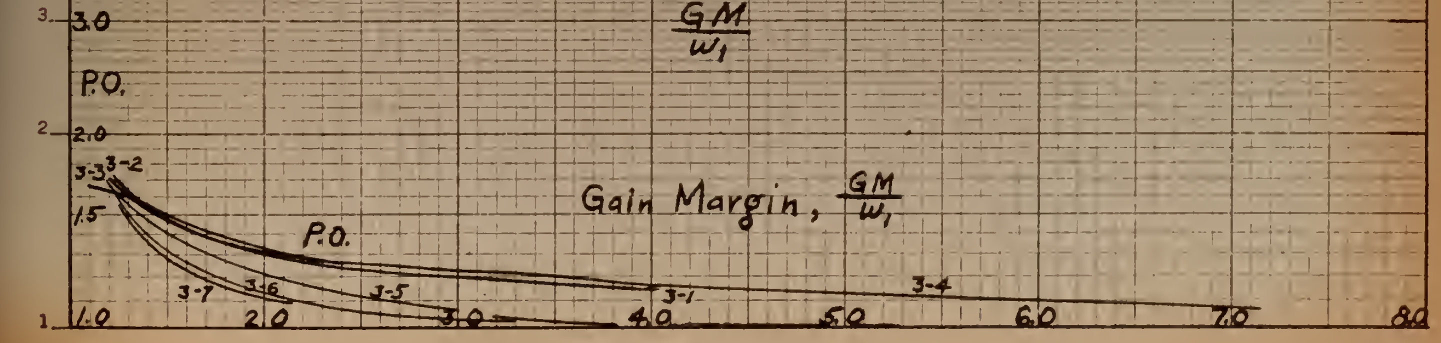


FIG.13

ϕ_M, M_p, and P.O. versus $\frac{GM}{w_1}$



In this report no systems incorporating parallel compensation in the feedback loop were considered. Systems having this type of compensation should be investigated to determine if the correlation of stability with ϕ_M can be made fully general.

As discussed in Appendix A, the experimental method could be much improved. A dynamic analyzer is desired having constant amplitude step and frequency outputs with a frequency range of 0.1 to 1000 cycles per second. If the analyzer measures accurately the phase shift and amplitude of the simulated system the experimental technique is simplified and the accuracy of measurements improved. In essence, then, a dynamic analyzer designed to operate over the optimum frequency range as determined by the analog computer is needed.

CHAPTER VI

CONCLUSIONS

In the design and adjustment of servo systems, the goal of the designer is to develop a system that will be stable and will meet certain specifications of accuracy. If the open-loop attenuation characteristic is selected, the phase shift characteristic of the system is defined. To determine if phase margin, the supplement of phase angle at zero decibels attenuation, could correlate dynamic and steady-state response was the purpose of this investigation.

It has been shown that a regular and ordered variation of Mp and P.O. with ϕ_M does exist. The correlation is displayed in Figs. 3 to 10 and an approximate linear relationship was shown. The relationship was made easier to use by plotting Mp and P.O. versus $\frac{K_v}{\omega_1}$, a dimensionless parameter, suffering a loss in generality thereby.

To the extent covered by this report, the correlation exists for systems having characteristic equations up to the fifth order and should be equally consistent for systems of all orders. The one ambiguity exists when a given ϕ_M may exist with two or more different system gain settings. In this case Mp is still related to P.O. by the equation derived by elimination of ϕ_M from equations (11) and (12).

Future investigation must incorporate other system parameters to effectively correlate other dynamic and steady-state characteristics with phase margin. A dynamic analyzer with a frequency range of 0.1 to 1000 cycles per second used with the electric analog computer can simplify measurement techniques.

Correlation of stability with phase margin should be of great assistance to the designer in developing linear automatic control systems.

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APPENDIX A

EXPERIMENTAL EQUIPMENT

The experimental system is shown schematically in Fig. 2. The Boeing Analog Computer BOAC was used to simulate the servo system. Automatic Balance Units were not available but drift gave difficulty only with higher gain systems in which voltage scaling was necessary. It is desirable that two persons work together when using the computer. Set-ups are more quickly checked for mistakes, computer checks performed and critical coefficients determined. There is no substitute for good computer technique. Being pressed for time, an effort was made to cut some corners. Partly as a result, an average of three test runs were made for each good test run. The troubles in most faulty runs were detected, but in a few the cause of the difficulty was not found and the computer set-up was redesigned. An example was inherent instability in a second order system at all gain settings—but a second order system should be absolutely stable. The limits recommended for system time constant components on the computer are $50K \leq R \leq 20M$ and $0.0001\mu f \leq C \leq 10\mu f$. In general the range should be somewhat more restricted, say, $0.1M \leq R \leq 10M$ and $0.001\mu f \leq C \leq 1\mu f$.

It was not possible to use the Mod. 100-A Dynamic Analyzer built by the Industrial Control Company since accurate readings of phase angle and magnitude cannot be made at low servo system frequencies.

The sine wave generator used was the Mod. 202-A Hewlett-Packard Low Frequency Generator. The sine wave is formed by clipping a triangular wave. At very low frequencies the irregularities of the wave shape were quite apparent, but the frequency components of the irregularities were of much higher order and small in amplitude and did not affect the response of low pass systems.

The step input was supplied by a 45 volt battery installed in the BOAC. If the battery voltage is low due to aging, it will be apparent on the brush recorder by abrupt random drifting in steady state.

The input and output signals were amplified by Brush amplifiers and recorded on paper with Brush magnetic pens. Much of the work done was in the range of 0.1 to 10 cps. The frequency response of the Brush magnetic pens is about:

0.2 cps--8mm peak to peak.
0.5 cps--10mm peak to peak.
80.0 cps--10mm peak to peak.

In measuring Mp, it is desirable that the recording instrument have a constant gain-frequency characteristic in the region of the resonant peak. The two pens used were not calibrated, but it was assumed their characteristics were essentially the same. This was confirmed by the fact that no measurable differance in result occured irregardless of which pen was recording output and the other input.

The resistors used were IRC, with one percent tolorence and temperature coefficient of 0.025 to 0.065. Sprague Vitamin Q Capacitors and Cornell-Dublier Metal Cased Dykanol paper capacitors were used. The Vitamin Q capacitors were determined to hold a charge better than the Dykanol capacitors. Polystyrene capacitors are much better than either of the above but were not available. The components were measured to four significant figures. It was assumed that these values did not change while in use.

The reader interested in servo system instrumentation and measurements is referred to F.C. Fickerisen and T.M. Stout [19], W.S. Pritchett and R.M. Saunders [20], R.J. Ehret [21] or F.H. Ferguson and C.H. Looney [22]. F.B. Anderson [23] described a ten cycle to 10 MC analyzer. To

work with the analogue computer a dynamic analyzer for measuring gain and phase angle through the frequency range of 0.1 to 1000 cycles is desirable.

APPENDIX B

DEVELOPMENT OF DATA

The systems tested in this report are listed and classified in Table I. In Table II are listed the values of ϕ_M, Mp, P.O., $\frac{K_v}{\omega_1}$ and $\frac{G.M.}{\omega_1}$ as determined by tests, plotting and computation. The method of plotting was stated in Chapter III and is common practice. The calculations, where used, were also routine, albeit tedius. The plotting of Mp and P.O. versus ϕ_M is apparently unique. Mp and P.O. vary in an exponential manner with ϕ_M. However, in the region of greatest interest $(40° \leq \phi_M \leq 60°)$, the curves may be approximated by straight lines, equations (11) and (12), introducing only small error. The curves are shown in Chapter IV, pages 16 to 23.

A more convenient method of using the data measured in this investigation is to plot Mp and P.O. versus $\frac{K_v}{\omega_1}$. Knowing only the corner frequencies of the amplitude characteristic, system gain may be selected to satisfy dynamic and accuracy requirements. If this is not possible, the designer knows at the very start of the design problem that further compensation is necessary. These curves are shown in Figs. 3 to 10 in Chapter IV, pages 16 to 23.

In Table III are listed ϕ_M, Mp, P.O., and G.M. for the higher order systems. Mp and P.O. versus ϕ_M are plotted in Fig. 14. Fig. 14 differs from Figs. 3 to 10 in that not only is $\frac{K_v}{\omega_1}$ varied but also the ratio and number of time constants.

In Fig. 11 Mp is plotted versus P.O. It is apparent that Mp has an ordered variation with P.O. The correlation is through their relationship with ϕ_M. Equation (14) was derived merely by eliminating ϕ_M in equations (11) and (12). This procedure turned out to be more accurate

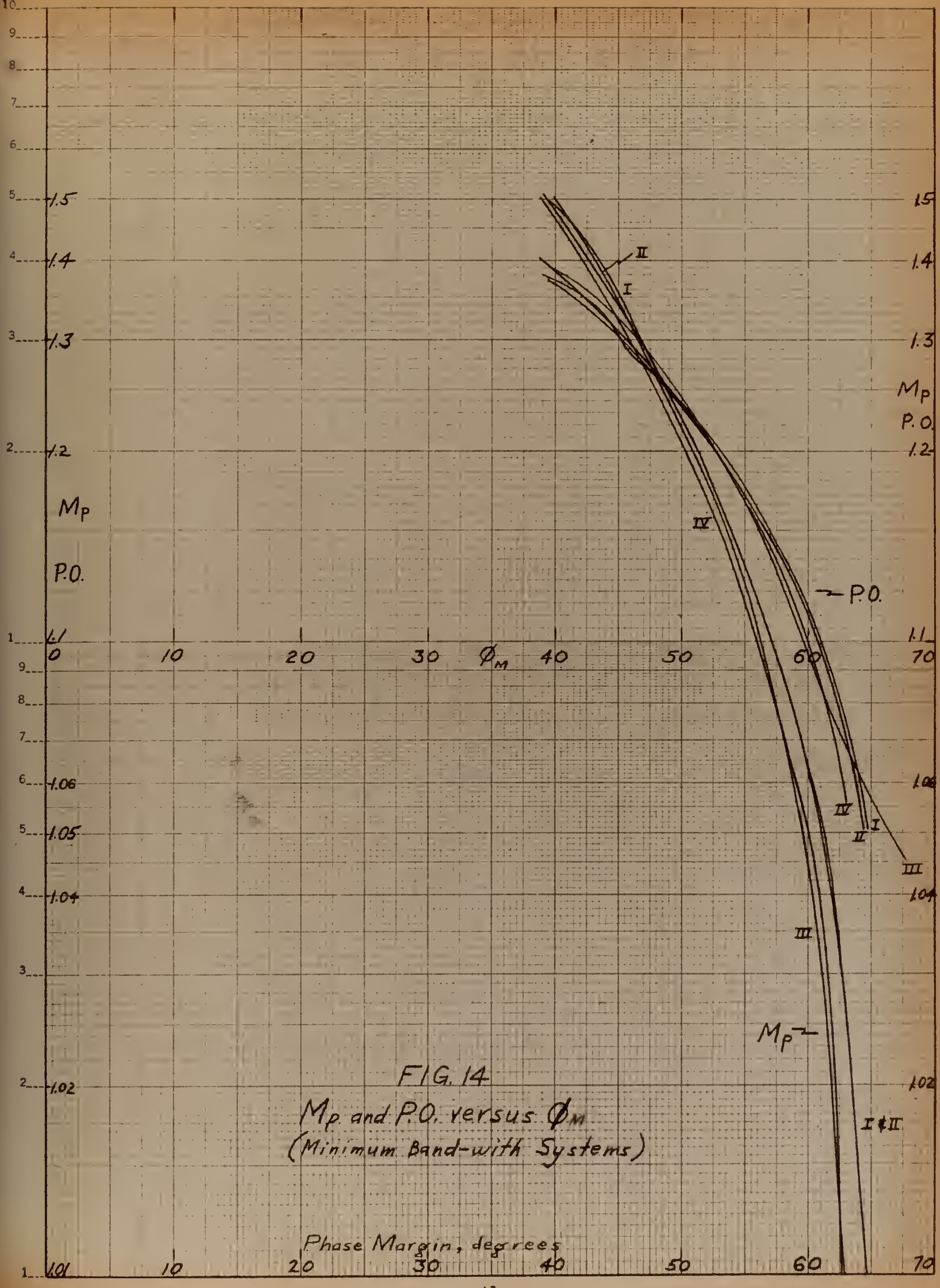

FIG. 14
M_p and P.O. versus ϕ_M
(Minimum Band-with Systems)

than any relationship that could be developed from Fig. 11.

The ratio of system gain to the lowest corner frequency of several representative systems is plotted versus ϕ_M in Fig. 12. It is apparent that the manner in which the system gain varies with ϕ_M cannot be so easily correlated as were Mp and P.O.

The ratio of gain margin to the lowest corner frequency is plotted versus ϕ_M for several conditionally stable systems in Fig. 13. Like in Fig. 12, a correspondence is apparent but it must vary significantly with other parameters as well as ϕ_M.

TABLE I

SYSTEMS TESTED

Second Order Characteristic Equation

Ident. No.	Type Zero	
2-1	$KG(j\omega) = \dfrac{K_v}{(j\omega T_1+1)(j\omega T_2+1)}$	, $\dfrac{T_1}{T_2} = 1$
2-2	Same	, $\dfrac{T_1}{T_2} = 10$
2-3	$KG(j\omega) = \dfrac{K_v(j\omega T_3+1)}{(j\omega T_1+1)(j\omega T_2+1)}$	, $\dfrac{T_1}{T_2}=1$, $\dfrac{T_1}{T_3}=25$
2-4	Same	, $\dfrac{T_1}{T_2}=5$, $\dfrac{T_1}{T_3}=10$
2-5	Same	, $\dfrac{T_1}{T_2}=5$, $\dfrac{T_1}{T_3}=50$
	Type One	
2-6.	$KG(j\omega) = \dfrac{K_v}{j\omega(j\omega T+1)}$	,
2-7	$KG(j\omega) = \dfrac{K_v(j\omega T_2+1)}{j\omega(j\omega T_1+1)}$	, $\dfrac{T_1}{T_2} = 25$
2-8	Same	, $\dfrac{T_1}{T_2} = 10$

TABLE I (continued)

Third Order Characteristic Equations
Type Zero

3-1	$KG(j\omega)=\dfrac{K_v}{(j\omega T_1+1)(j\omega T_2+1)(j\omega T_3+1)}$	, $\frac{T_1}{T_2}=10$, $\frac{T_1}{T_3}=100$
3-2	Same	, $\frac{T_1}{T_2}=5$, $\frac{T_1}{T_3}=50$
3-3	Same	, $\frac{T_1}{T_2}=1$, $\frac{T_1}{T_3}=10$

Type One

3-4	$KG(j\omega)=\dfrac{K_v}{j\omega(j\omega T_1+1)(j\omega T_2+1)}$	, $\frac{T_1}{T_2}=10$
3-5	Same	, $\frac{T_1}{T_2}=4.77$
3-6	Same	, $\frac{T_1}{T_2}=2.3$
3-7	Same	, $\frac{T_1}{T_2}=1$
3-8	$KG(j\omega)=\dfrac{K_v(j\omega T_2+1)}{j\omega(j\omega T_1+1)(j\omega T_3+1)}$	, $\frac{T_1}{T_2}=10$, $\frac{T_1}{T_3}=100$
3-9	Same	, $\frac{T_1}{T_2}=5$, $\frac{T_1}{T_3}=20$

Fourth Order Characteristic Equations
Type One

4-1	$KG(j\omega)=\dfrac{K_v(j\omega T_2+1)}{(j\omega)(j\omega T_1+1)(j\omega T_3+1)^2}$	, $\frac{T_1}{T_2}=5$, $\frac{T_1}{T_3}=100$
4-2	$KG(j\omega)=\dfrac{K_v(j\omega T_2+1)^2}{j\omega(j\omega T_1+1)^2(j\omega T_3+1)}$	, $\frac{T_1}{T_2}=8.3$, $\frac{T_1}{T_3}=100$

TABLE II

TEST DATA

System	K_v/ω_1	ϕ_M, deg.	M_p	P. O.	G.M./ω_1
2-1	.158	67.5	—	1.17	Abs.
	.281	49.5	1.19	1.28	Stable
	.398	40.5	1.44	1.33	
	.50	36.0	1.58	1.37	
	.63	32.0	1.8	1.41	
	.89	26.0	2.24	1.475	
	1.58	20.0	2.98	1.57	
2-2	5.0	59.0	1.02	1.144	Abs.
	7.93	47.5	1.24	1.23	Stable
	10.0	42.5	1.38	1.276	
	15.8	33.5	1.76	1.366	
	25.2	26.5	2.24	1.475	
	31.6	24.0	3.51	1.497	
	50.0	19.0	3.12	1.576	
2-3	0.25	56.5	1.07	1.0003	Abs.
	.5	43.0	1.38	1.17	Stable
	1.0	35.0	1.68	1.238	
	1.58	31.5	1.88	1.36	
	2.5	30.0	2.0	1.397	
	5.0	31.5	1.94	1.412	
	10.0	37.0	1.78	1.39	
	15.8	43.0	1.59	1.365	
	25.0	51.5	1.46	1.318	
	50.0	64.0	1.26	1.278	
	100.0	75.0	1.16	1.224	

TABLE II (continued)

System	K_v/ω_1	ϕ_m, deg.	M_p	P.O.	$G.M./\omega_1$
2-4	3.98	89	0.978	1.002	Abs. Stable
	10.0	75.5	0.994	1.034	
	20.0	75.0	1.005	1.053	
	50.1	75.0	1.034	1.052	
	100.0	77.5	1.034	—	
	316.0	82.5	1.022	—	
2-5	10.0	52.0	1.147	1.166	Abs. Stable
	25.1	39.0	1.53	1.278	
	50.1	35.5	1.68	1.324	
	100.0	36.0	1.72	1.328	
	251.0	44.5	1.53	1.286	
	501.0	55.0	1.36	1.228	
2-6	.565	63.0	1.0	1.055	Abs. Stable
	1.0	51.0	1.165	1.162	
	1.76	40.0	1.436	1.279	
	3.16	30.0	1.96	1.398	
	5.61	22.5	2.66	1.507	
	10.0	17.5	3.61	1.605	
2-7	1.0	54.0	1.15	1.237	Abs. Stable
	1.58	46.0	1.29	1.29	
	2.51	39.0	1.53	1.35	
	3.98	34.0	1.74	1.41	
	6.3	30.0	1.93	1.46	
	10.0	28.5	2.11	1.5	
	15.8	28.0	2.18	1.53	
	25.1	29.0	2.14	1.542	
	39.8	30.0	2.02	1.537	

TABLE II (continued)

System	K_v/ω_1	ϕ_M, deg	M_p	P.O.	$\frac{G.M.}{\omega_1}$
2-7 (cont.)	63.0	34.0	1.88	1.518	Abs.
	100.0	38.5	1.74	1.483	Stable
	158.0	44.0	1.57	1.441	
	250.5	51.0	1.43	1.383	
2-8	1.0	57.0	1.10	1.129	Abs.
	2.0	45.0	1.34	1.232	Stable
	5.01	36.0	1.64	1.337	
	10.0	34.0	1.8	1.37	
	25.16	37.0	1.74	1.356	
	50.1	45.0	1.53	1.312	
	100.0	55.5	1.35	1.25	
	200.0	67.5	1.22	1.177	
	398.0	77.0	1.134		
3-1	7.95	60.0	1.01	1.145	3.94
	12.6	47.0	1.26	1.256	2.37
	20.0	35.0	1.66	1.386	1.73
	31.6	23.0	2.51	1.536	1.41
	50.1	13.0	4.47	1.677	1.15
3-2	6.31	52.0	1.15	1.24	2.64
	10.0	38.0	1.513	1.36	1.84
	15.8	25.0	2.37	1.47	1.47
	25.1	15.0	3.8	1.69	1.28
3-3	2.51	70.0	0.9	1.22	2.61
	3.98	49.0	1.26	1.356	1.81
	5.01	35.0	1.66	1.438	1.62
	6.31	31.0	1.885	1.504	1.465
	10.0	19.0	3.09	1.63	1.272

TABLE II (continued)

System	K_v/ω_1	ϕ_M, deg.	M_p	P.O.	G.M./ω_1
3-4	0.561	61.0	1.022	1.06	8.50
	0.63	58.0	1.034	1.10	7.56
	0.793	52.0	1.15	1.15	6.00
	1.0	47.0	1.242	1.21	4.77
	1.26	41.0	1.41	1.26	3.80
	1.77	34.5	1.716	1.36	2.68
	2.0	28.0	2.09	1.40	1.74
	3.16	21.0	2.85	1.55	1.56
	4.14	15.0	3.76	1.63	1.293
3-5	0.398	66.0	1.0	1.0022	5.28
	0.63	55.0	1.1	1.093	2.86
	1.0	42.5	1.38	1.23	1.916
	1.58	30.0	2.0	1.39	1.52
	2.5	19.0	3.02	1.58	1.30
3-6	0.316	66.0	1.0	1.03	3.16
	0.5	55.0	1.1	1.11	2.07
	0.707	45.5	1.3	1.23	1.67
	1.0	34.5	1.70	1.36	1.44
	1.58	20.5	3.02	1.54	1.26
3-7	0.316	57.5	1.05	1.10	2.067
	0.398	52.0	1.15	1.163	1.78
	0.5	45.0	1.33	1.25	1.58
	0.561	40.0	1.51	1.29	1.505
	0.63	36.0	1.7	1.33	1.44
	0.75	30.0	1.99	1.43	1.315
	1.0	20.0	2.98	1.55	1.26

TABLE II (continued)

System	K_v/ω_1	ϕ_M, deg.	M_p	P.O.	G.M./ω_1
3-8	1.0	55.0	1.11	1.103	Abs. Stable
	2.0	44.0	1.35	1.254	
	5.01	35.5	1.66	1.352	
	12.6	34.0	1.80	1.37	
	25.1	35.0	1.80	1.37	
	50.1	40.0	1.62	1.346	
	100.0	49.0	1.44	1.288	
	158.0	53.0	1.35	1.252	
	251.2	56.0	1.27	1.214	
	398.0	56.0	1.22	1.17	
	631.0	50.5	1.20		
	1000.0	44.5	1.35		
	2000.0	33.5	1.72		
	3160.0	27.0	2.11		
3-9	1.58	53.0	1.148	1.184	Abs. Stable
	3.16	44.0	1.38	1.257	
	6.31	40.0	1.51	1.296	
	12.6	40.0	1.515	1.32	
	25.1	42.0	1.51	1.272	
	50.1	41.0	1.483	1.224	
	100.0	35.0	1.66	1.323	
	200.0	27.5	2.11	1.429	
4-1	10.0	48.5	1.2	1.18	79.5
	100.0	55.0	1.25	1.23	7.95
	316.0	34.0	1.77	1.36	2.52
4-2	159.0	0.0	Unstable		
	1000.0	32.0	1.9	1.45	
	10,000.0	37.5	1.54	1.4	

TABLE III

MINIMUM BAND-WIDTH SYSTEMS [12]

Case I: $KG(j\omega) = \dfrac{K_v(j\omega T_1+1)^2}{(j\omega)^3(j\omega T_2)}$

ϕ_m, deg.	M_p	P.O.	GM, db
64.6	1.01	1.053	52.2
61.2	1.05	1.099	38.9
57.0	1.1	1.148	33.7
51.2	1.2	1.223	29.3
46.6	1.3	1.287	26.6
43.5	1.4	1.334	26.2
39.4	1.5	1.377	25.6

Case II: $KG(j\omega) = \dfrac{K_v(j\omega T_1+1)}{(j\omega)^2(j\omega T_2+1)}$

ϕ_m, deg.	M_p	P.O.	GM, db
64.5	1.01	1.052	Abs
61.0	1.05	1.097	Stable
57.2	1.1	1.145	
51.1	1.2	1.22	
46.6	1.3	1.28	
42.5	1.4	1.33	
39.4	1.5	1.37	

Case III. $KG(j\omega) = \dfrac{K_v(j\omega T_1+1)^2}{(j\omega)^3(j\omega T_2+1)^2}$

62.9	1.01	1.055	51.8
59.6	1.05	1.099	38.7
56.4	1.1	1.147	33.2
50.9	1.2	1.23	28.3
46.5	1.3	1.30	25.8
43.1	1.4	1.35	24.0
39.9	1.5	1.38	22.9

$KG(j\omega) = \dfrac{K_v(j\omega T_1+1)}{(j\omega)^2(j\omega T_2+1)^2}$

62.6	1.01	1.06	-17.9
60.0	1.05	1.10	-17.4
56.0	1.1	1.15	-16.8
50.2	1.2	1.235	-15.8
45.8	1.3	1.30	-14.9
42.0	1.4	1.35	-14.1
38.9	1.5	1.40	-13.3

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